



Relay Test – Round 11003

mathleague.org – Relay Round 11003.1

(1-1) Three marbles are in a bag, each of which is equally likely to be black or white. A white marble is added to the bag. If a marble is removed from the bag, what is the probability that it is black?

(1-2) Let $T = \text{TNYWR}$. Evaluate $\cos(\text{Arctan}(2T))$.

(1-3) Let $T = \text{TNYWR}$. What is the sum of the squares of the roots of $x^3 + 4x^2 + 6x + T$?

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(2-1) What is the area of the largest square that can be inscribed in a circle of area 100π ?

(2-2) Let $T=TNYWR$. What is the area of an equilateral triangle with side length T ?

(2-3) Let $T=TNYWR$. Define the “Tribonacci Sequence” as $T_1=1$, $T_2=1$, $T_3=2$, and for $n > 3$,
 $T_n = T_{n-1} + T_{n-2} + T_{n-3}$. Within every subsequence consisting of eight consecutive terms of the Tribonacci sequence, there are exactly x terms divisible by T . Find x .

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(3-1) Eight people sit in a row of chairs, but the oldest three must sit in three consecutive seats. How many such seating arrangements are possible?

(3-2) Let $T=TNYWR$, and let $R = T/36$. R is the sum of the first n triangular numbers (i.e. R is the n th tetrahedral number). What is n ?

(3-3) Let $T=TNYWR$. Find the T th digit after the decimal point in the decimal expansion of π .

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(4-1) Let n be a 24-digit base-2 number with all digits equal to 1. Find $\log_8(n+1)$.

(4-2) Let $T=TNYWR$ and let S be the cube of T . Compute the smallest divisor of $S-1$ that is greater than 1.

(4-3) Let $T=TNYWR$. If $k = \frac{a}{T} + \frac{T}{a}$, and a and T are relatively prime integers, compute the smallest possible value of k .

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(5-1) Find the sum of the first three triangular numbers.

(5-2) Let $T=TNYWR$. Convert the base T number 22 into a base ten representation.

(5-3) Let $T=TNYWR$. What is the smallest positive integer that has exactly T factors?



Relay Test – Round 11003 (Solutions)

Answers for Question 3 must be exact and simplified.

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|-------|-------|-------|-----------------|-------|---------------|
| (1-1) | $3/8$ | (1-2) | $4/5$ | (1-3) | 4 |
| (2-1) | 200 | (2-2) | $10000\sqrt{3}$ | (2-3) | 0 |
| (3-1) | 4320 | (3-2) | 8^* | (3-3) | 5 |
| (4-1) | 8 | (4-2) | 7 | (4-3) | 113/56 |
| (5-1) | 10 | (5-2) | 22 | (5-3) | 3072 |

* Knowledge of tetrahedral numbers is fair game for future contests.